Role of Jet Stability in Edgetone Generation

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Theme

AN edgetone is the discrete tone or narrow-band sound produced by an oscillating free shear layer (e.g., a jet, wake, or a separated boundary layer) impinging on a rigid surface (the edge). The edgetone phenomenon is not only of fundamental fluid mechanical interest, but has practical implications in such diverse applications as wind-driven musical instruments and sirens, fluidic oscillators, noise from cavities in moving vehicles and noise from ventilated transonic wind-tunnel walls. In this paper we are concerned with jet edgetones only. Although the oscillatory fluid motion involved in the edgetone must be governed by the stability characteristics of the flowfield, the classical theory for the propagation of small oscillations in a parallel shear flow does not explain the major observed features of the edgetone. The objective of the present paper is to briefly note the major shortcomings of this theory for the edgetone, and show that these can be explained satisfactorily on the basis of nonparallel flow stability theory. The major ideas and implications of a theory for stability of nearly-parallel flows (developed in Ref. 1) is extracted from Ref. 2. The features of propagating disturbances in such flow fields are strikingly similar to those of the edgetone. Postulation that the edgetone frequency is simply that of the disturbance receiving the greatest amplification by the developing flowfield yields qualitative experimental and theoretical accord for the edgetone. Quantitative comparisons are currently under way.

Contents

The main features of edgetone operation are the following. With a given jet-edge system, the frequency f depends on the jet characteristic speed U_m at the nozzle exit and the distance h (referred to as the breadth) between the exit and the edge. There are several stages of operation. For each stage of operation, there is a minimum breadth $h_{\min}$ at a given U_m, and a minimum speed $U_{\min}$ at a given h. In any stage, the frequency is roughly proportional to U_m and inversely proportional to h. Observations showing that the frequencies of edgetones are all within the range of frequencies which receive amplification in the jet flow, that the frequency increases with U_m, and that the tone generated is characterized by features such as the minimum breadth. These suggest that the stability of the jet in a jet-edge system is likely the principal agency in initiating an edgetone and perhaps, in controlling its main features. A survey of modern experimental and analytical studies on the jet edgetone may be found in Ref. 3 where other references are cited.

In the present study the jet edgetone features are examined in the light of hydrodynamic stability theory of an incompressible, two-dimensional jet subjected to small time-dependent disturbances. First, we consider a parallel jet where the velocity field is given by $\mathbf{U} = U(y)\mathbf{i}$, being in the streamwise direction, x, and y is normal to it. The disturbances are considered to be travelling harmonic waves propagating in the streamwise direction, and are represented by a stream function $\psi(x, y, t)$ expressed as

$$\psi = \phi(y)\, e^{i(\alpha x - \omega t)} = \phi(y)\, e^{-\alpha_i x}\, e^{i\alpha_r(x-ct)}$$

where ω, $\alpha = \alpha_r + i\alpha_i$, and $c = \omega/\alpha_r$ are constants. Here, ω, assumed real, is the frequency; α_i denotes the rate of spatial exponential amplification of the disturbance; and c is the phase or wave speed.

In the linear stability theory with which we are concerned here, for a given mean velocity profile $\bar{U}(\eta) = U/U_m$, where $\eta = y/\delta$, δ being a characteristic thickness of the jet shear layer (such as the momentum thickness), there results an eigenvalue problem in the quantities $\bar{\omega} = \omega\delta/U_m$, $\bar{\alpha} = \alpha\delta$, and R, the Reynolds number $= U_m\delta/\nu$, ν being the kinematic viscosity of the fluid. Typical solutions of such a problem for a given mean velocity profile are similar to those illustrated in Fig. 1, where the $\omega =$ the constant curves (which will be discussed later) should be disregarded at the moment. In such an $\bar{\omega} - R$ plot, the path of a disturbance of given frequency ω down a parallel jet of given R is simply a point.

Through a given parallel jet, a disturbance will grow at the rate, $e^{-\alpha_i x}$, and its phase will change linearly at the rate $\alpha_r x$, where α_i and α_r as noted before are constants. One frequency, ω_m, the dominant frequency will receive the greatest amplification (i.e., be associated with the most negative value of α_i). A disturbance of discrete frequency appearing naturally in this flowfield might be expected to have the frequency ω_m. If the edgetone system is operating at a given frequency, which might be presumed to be ω_m, and the edge is moved, the frequency is no longer ω_m. Thus, some auxiliary criterion is necessary to explain the edgetone frequency selection process. Measurements of the spatial amplification and phase change rates for the edgetone disturbances[3] also show clearly that the effective α_i and α_r are not

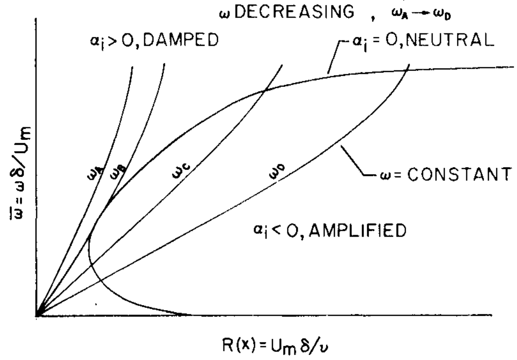

Fig. 1 Stability characteristics of a jet.

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Index categories: Boundary-Layer Stability and Transition; Jets, Wakes, and Viscid-Inviscid Flow Interaction; Aircraft Noise Aerodynamics (Including Sonic Boom).

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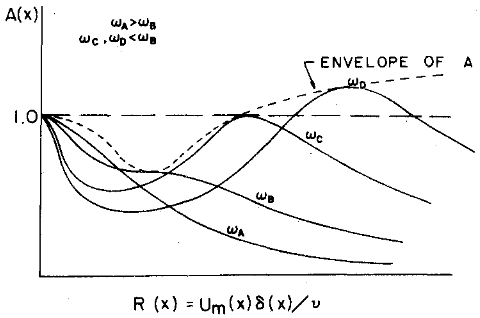

Fig. 2 Amplitude histories of disturbances in a nonparallel jet.

constant along the jet. Thus, parallel flow stability theory does not hold the complete explanation of the edgetone phenomenon.

Consider now a nonparallel jet with the mean velocity field being described by $\mathbf{V}(x, y) = U(x, y)\mathbf{i} + V(x, y)\mathbf{j}$, where $\mathbf{j}$ is the direction of the y axis. We now seek $\psi(x, y, t)$ as a traveling harmonic wave propagating in the x direction but with varying spatial attenuation rate and phase speed. Thus, it may be expressed as

$$\psi(x, y, t) = \phi(x, y)\, e^{i[\theta(x) - \omega t]}; \quad \theta(x) = \int [\alpha_r(x) + i\alpha_i(x)]\, dx$$

where ω, assumed real, is the frequency; $\alpha_i(x)$ may be referred to as the local spatial amplification rate, and $\alpha_r(x)$ as the local wave number. The local wave speed is given by $c(x) = \omega/\alpha_r(x)$.

In the linear theory[1,2] of the stability of a slightly nonparallel jet, the solution for the eigenvalues of $\bar{\omega} = \omega\delta(x)/U_m(x)$, $\bar{\alpha} = \alpha(x)\delta(x)$, $R = U_m(x)\delta(x)/v$, for a given jet profile $\bar{U} = U[x, \eta(x)]/U_m(x)$ where $\eta(x) = y/\delta(x)$, are, at each streamwise position x, the same as those for a parallel jet with mean velocity profile whose dependence on η is identical to that of the local $\bar{U}(x, \eta)$ in the nonparallel flow, and with a Reynolds number equal to the local Reynolds number $R(x)$ in the nonparallel flow.

A map of the stability characteristics of a given nonparallel jet is illustrated in Fig. 1 where now $\bar{\omega}$, $\bar{\alpha}$, and $\bar{R}$ are all functions of x.

In the $\bar{\omega}(x) - R(x)$ plane, the path of a disturbance of fixed frequency ω propagating in the nonparallel flow is described by $\bar{\omega}(x) = \omega v R(x)/U_m{}^2(x)$, as is illustrated in Fig. 1 by $\omega_A - \omega_D$.

As the disturbance propagates through the jet, its propagation features; namely, $\alpha_i(x)$ and $c(x)$ for a given ω, $U_m(x)$, and $\delta(x)$ can be determined by evaluating $\bar{\alpha}_i(\bar{\omega}, R)$ and $\bar{c}(\bar{\omega}, r)$ along its path in the $\bar{\omega} - R$ plane.

The ratio of amplitudes for a disturbance of given frequency between two stations in the flowfield, $A(x)$, may be obtained by integration of $\alpha_i(x)$ along the appropriate path for that frequency. Examples of such resultant amplitude histories are depicted in Fig. 2. The amplification rates are not pure exponentials. We note that the amplitude histories form an envelope describing the maximum growth of disturbances up to a given x corresponding to $R(x)$. At the point of tangency of its amplitude history to the envelope, a disturbance becomes the dominant disturbance in the flow (assuming equal excitation). Its frequency is $\omega_d(x)$. A continuous decrease in ω_d as x increases is indicated by the theory.

The phase variation may be similarly determined by integration of $\alpha_r(x)$ along the path. These stability characteristics of a nonparallel jet reflect satisfactorily the main observed features of edgetone disturbances. It is proposed that if an edgetone is generated, the frequency of the tone is the frequency of the disturbance which receives the maximum total amplification over the distance h and that a tone is generated only if the total amplification ratio over the distance h for some disturbance is equal or greater than unity.

On the basis of these criteria and Fig. 2 constructed for a jet flow as it exists in the given edgetone situation, we can conclude the following with regard to some of the main features of the edgetone. 1) No tone can be naturally generated when the envelope of $A(h)$ is less than unity. 2) It follows that for a given jet speed, U_{mo}, at the jet nozzle exit, there is a minimum value of h for initiation of an edgetone. 3) Similarly, for a given h there is a minimum speed. 4) The edgetone frequency ω_d, will increase with U_m and decrease with h. For further details, see Refs. 1, 2, and 4.

Studies of the type described here, it is believed, will throw light not only on the role of stability of shear layers in various types of edgetones and related problems of aerodynamic noise generation, such as noise from the coherent structure of jets, but also on the basic features of stability in many problems of transition and turbulence.

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